The similar universal behavior of Gravitational Wave Signal between Single Rotating Neutron Star and binary neutron stars

TAUP2019

Ng Ho Yin (Harry), Cheong Chi Kit (Patrick)
Dr. Lin Lap Ming
Prof. Tjonnie, G. F. Li

10/9/2019

香港中文大學
The Chinese University of Hong Kong

Outline

- Introduction
- Objectives
- Methods
- Results
- Conclusion

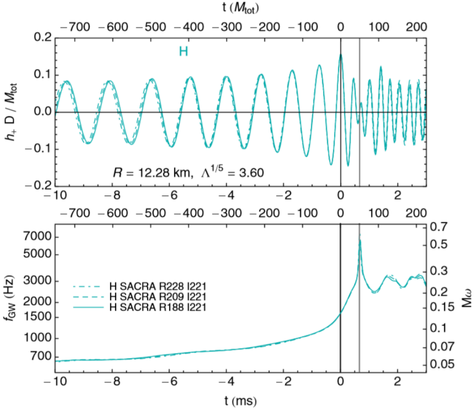
Introduction

- A large amount of Studies of Spectral properties of GW signal from Binary Neutron stars (BNS) of Inpiral, Merger and postmerger (Luciano Rezzolla et al. (2016), Shibata et al. (2013))
- Constrains Radius, Compactness, Tidal Deformability, e.g. Mass Constraint of Hypermassive NS from Observaing GW singal of BNS (GW170817)
- Unknown Equation of state (EOS) of NS
  `How to Study them?`
- Highly Non-linear problems and difficult to observe
- Gamma-Ray Bursts(GRBs) and GW signals from BNS (Before Construction of KAGRA and Ad. LIGO)
- Numerical Simulation!

Motivation

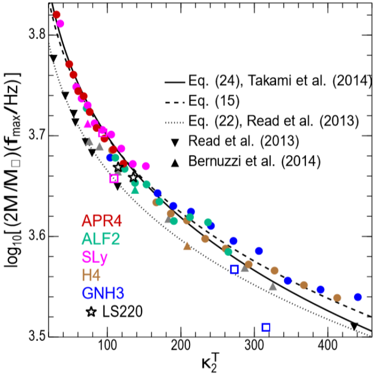
`Popular Simulation Results (Baiotti+ 2008, Stergioulas+ 2011, Takami 2014, Shibata+ 2013, Rezzolla+ 2014 etc.)`

- Inspiral Phase
- The instantaneous freq. at GW amplitude's max. f_{max} (Time of Merger) (Jocelyn S. Read, et al. 2013)



Motivation

- Inspiral Phase
- Freq.at GW amplitude's max. f_{max}, Qusai-Universality relations between f_{max}, dimensionless tidal Love Number of Single non-rotating NS K_2^T
- Different EOSs almost obey the same trend



(Rezzolla et al. (2018))

Objectives

We bravely assume f_{max} is related to some pulsation freq. modes of single oscillating NS

- Study the appearence of such quasi-univseralit y in Single isolated Rotating NSs PULSATION FREQUENICES!
- If appears $=>$ Regarding of BNSs, RNSs, NSs (It must relate to the fundamental properties of NS matter)
- Making a tight Constraints on the EOS
- Prove the f_{max} and PULSATION FREQUENICES have sth in common

Pulsation modes of Single Oscillating and rotating NS

- Perturbation effects
- NS is oscillating => different oscillation modes (pulsation modes)
- High freq $1000 - 10000Hz$
- Features of different modes depends on different properties of NS, like rotation, EOS, metric dynamics, etc.

  Perturbations due to
- NS formed just after Core-Collapse Supernova (CCSNe)
- Starquakes
- Hypermassive neutron star (HMNS) formed after merger
- etc.

Methods

All the values in code units $c = M_{\odot} = G = 1$

1. Initial Models with different EOSs

Simulate Initial Models with different Gravitational Masses and EOSs, with the same uniform angular velocity $\Omega_c = 1.293 * 10^{-2}$ (around $2600 rads^{-1}$)

2. Adding perturbation to the initial data and evolve the initial models with Multi-D simulation code - **Gmunu**

3. Plotting the oscillation velocity profile against time and extract the different pulsation modes by Fourier Transform

Methods

4. Using a TOV solver to calculate the Love number of corresponding static initial models (as tidal deformability defined by static NS)

5. Finding which pulsation modes have similar quasi-universality relation as same as produced by BNS f_{max} and Love number Λ

Initial Model - EOS

Love Number Λ - Star's quadrupole deformation in response to the companion's perturbing tidal field $-- >$ closely related to the EOS
EOS - All about matters, Equations of state are useful in describing the properties of fluids, mixtures of fluids, solids, and the interior of stars.

- Polytropic EOS with $P = K\rho^{\gamma}$, $\gamma = 2$ and $K = 100$
- Realistic EOSs is made by piecewise Polytropic EOS (Jocelyn S. Read et al. 2009)
- Assume Perfect Fluid $T^{\mu\nu} = \rho h u^{\mu} u^{\nu} + P g^{\mu\nu}$
- Initial Perturbation to excite $l = 2$ pulsation mode $v_{\theta} = a \sin\left(\pi \frac{r}{r_{(\theta)}}\right) \sin\theta \cos\theta$

Simulation Code – **Gmunu**– Hydro

The 3+1 "Valencia"formulation

$$\partial_t(\sqrt{\gamma}\vec{U}) + \partial_i(\sqrt{\gamma}\vec{F^i}) = \vec{S}$$

where

$$\boldsymbol{U} = \begin{bmatrix} D \\ S_j \\ \tau \end{bmatrix} = \begin{bmatrix} \rho W \\ \rho h W^2 v_j \\ \rho h W^2 - p - D \end{bmatrix}$$

$$\boldsymbol{F}^i = \begin{bmatrix} D\left(v^i - \beta^i/\alpha\right) \\ S_j\left(v^i - \beta^i/\alpha\right) + \delta^i_j D \\ \tau\left(v^i - \beta^i/\alpha\right) + P v^i \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 \\ T^{\mu\nu}\left(\frac{\partial g_{\nu j}}{\partial x^\mu} - \Gamma^\lambda_{\mu\nu} g_{\lambda j}\right) \\ \alpha\left(T^{\mu 0}\frac{\partial \ln\alpha}{\partial x^\mu} - T^{\mu\nu}\Gamma^0_{\mu\nu}\right) \end{bmatrix}$$

Simulation Code – **Gmunu**– Metric

Conformal Flatness Approximated Metric

$$g_{\mu\nu} = \begin{bmatrix} -\alpha^2 + \beta_i\beta^i & \beta_1 & \beta_2 & \beta_3 \\ \beta_1 & \psi^4 & 0 & 0 \\ \beta_2 & 0 & \psi^4 r^2 & 0 \\ \beta_3 & 0 & 0 & \psi^4 r^2 \sin^2\theta \end{bmatrix}$$

Simulation Code – **Gmunu**– Metric Evolution

XCFC scheme

$$\tilde{\Delta} X^i + \frac{1}{3}\tilde{\nabla}^i\left(\tilde{\nabla}_j X^j\right) = 8\pi\tilde{S}^i$$

$$\tilde{A}^{ij} \approx \tilde{\nabla}^i X^j + \tilde{\nabla}^j X^i - \frac{2}{3}\tilde{\nabla}_k X^k f^{ij}$$

$$\tilde{\Delta}\psi = -2\pi\tilde{E}\psi^{-1} - \frac{1}{8} f_{ik} f_{jl}\tilde{A}^{kl}\tilde{A}^{ij}\psi^{-7}$$

$$\tilde{\Delta}(\alpha\psi) = (\alpha\psi)\left[2\pi\left(\tilde{E} + 2\tilde{S}\right)\psi^{-2} + \frac{7}{8} f_{ik} f_{jl}\tilde{A}^{kl}\tilde{A}^{ij}\psi^{-8}\right]$$

$$\tilde{\Delta}\beta^i + \frac{1}{3}\tilde{\nabla}^i\left(\tilde{\nabla}_j\beta^j\right) = 16\pi\alpha\psi^{-6} f^{ij}\tilde{S}_i + 2\tilde{A}^{ij}\tilde{\nabla}_j\left(\alpha\psi^{-6}\right)$$

Simulation Code – **Gmunu**– Key Features

Gmunu (General-relativistic multigrid numerical Einstein solver)

- 3+1 in Spherical polar coordinates (1,2,3-D)
- Hydro: high-resolution shock-capturing (HRSC) methods
  - Reconstruction method: Piecewise-Constant (PC), TVD, (5-th order)WENO, MP5
  - Riemann solver: HLL, HLLE, Marquina
  - Time update: RK3 methods
  - Conserved variables to Primitive variables: Regula-Falsi method
- Conformally flatness condition(CFC) metric evolution:
  - extended CFC (xCFC) scheme (Cordero-Carrion et al. (2009))
  - Multigrid solver for the elliptic non-linear coupled equations

Details would be presented by my another groupmate - Patrick in later talk

Simulation SETUP

- Grid: $320 \mathrm{x} 64$
- Atmosphere density $\approx 10^{-6}$ of ρ_c
- At least $10ms$ evolution of the initial models

Time limitation

- The work of Simulation code **Gmunu** are done in the last month
- The simulation results are still running for realistic EOSs
- Only have Polytropic results ($\gamma = 2$, $K = 100$)

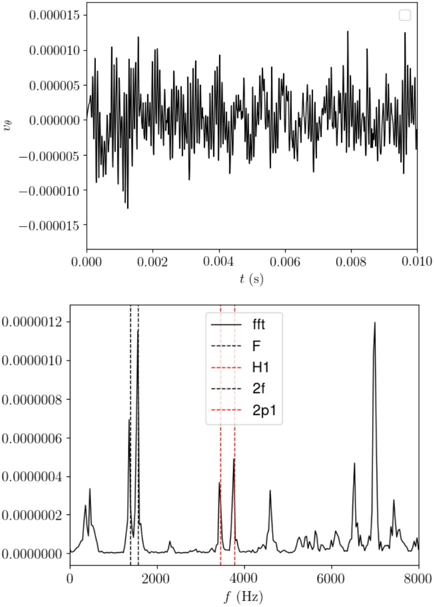
Model

All the rotating initial NS are with Uniform angular velocity $\Omega = 1.293\text{x}10^{-2} = 2640rads^{-1}$

Tabela: $\Gamma = 2,\ K = 100, (c = G = M_{sun} = 1)$.

Model	Rotating NS $M_\odot$	Λ of Corresponding Static NS
1	1.40	744.811
2	1.30	1428.61
3	1.28	1928.68
4	1.24	2246.20

Results


0.000015
0.000010
0.000005
0.000000
−0.000005
−0.000010
−0.000015
v_θ
0.000
0.002
0.004
0.006
0.008
0.010
t (s)
0.0000012
0.0000010
0.0000008
0.0000006
0.0000004
0.0000002
0.0000000
fft
F
H1
2f
2p1
0
2000
4000
6000
8000
f (Hz)

Different pulsation modes

- Model 1
- F (radial mode), ${}^2f$ (fundamental of non-radial mode, i.e. $l = 2$), H_1 (first overtone of radial mode), ${}^2p_1$ (first overtone of non-radial mode)
- F, ${}^2f$ are in the range of $1000 - 2000Hz$
- Freq range of the instantaneous freq of the GW max. amplitude f_{max} in BNS case
- Dimensionless tidal Love numbers $\kappa_2^T = \frac{3}{16}\Lambda = \frac{3}{16}\frac{\lambda}{M^5}$ (Rezzolla+, 2016)

Results

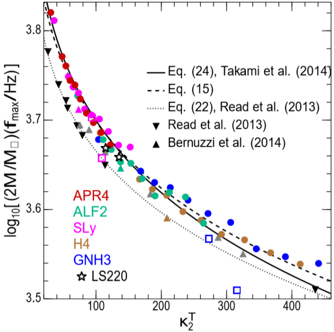


Figura: BNS case, f_{max} is the instantaneous freq at GW with max amplitude

Results

Fitting Line is the black dash line (Rezzolla+,2016)

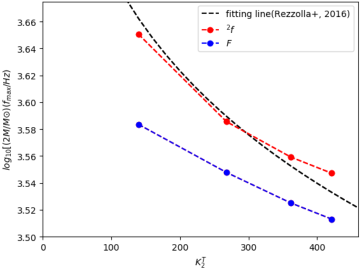
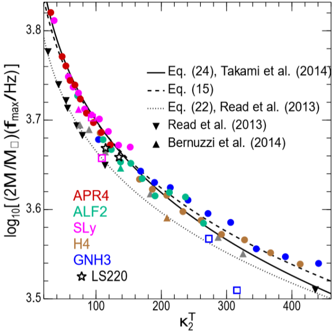
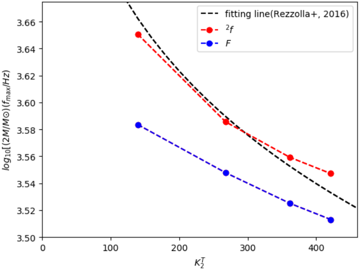


Figura: Single Isolated Rotating NS case with 2 pulsation freq.

Results

Fitting Line is the black dash line (Rezzolla+,2016)





Results

- Though Only Polytopric case
- ${}^2f$ is the answer!
- Lower Mass $->$ Diverage, just like BNS case
- Due to Divergence, the low-mass cases have been excluded in this relations

Reasons of low mass models diveragence

1. Non-deformed initial BNS for simulation from Rezzolla+,2016 $->$ BNS with higher deformabilities binaries would have higher errors in sim.
2. Objects with low enough deformability (hard enough EOSs) would have such relations $->$ exclude the low-mass models and make constraints on the EOS.

Conclusion

- BNS **VS** NS
- ${}^2f$ has higher possibility related to f_{max}
- Common point of between single RNS and BNS GW signal freq is found!
- f_{max} vs Λ relation is not unique for BNS but also RNS NS
- With Constraints of Λ, like B. P. Abbott et al. 2017 (One of the yesterday talks)
- Make new constraints on Single Rotating NSs

Future Works

- More initial models with concentrated in high mass NS
- Check whether the running simulations of realistic EOSs models also obey it.
- Different Angular velocity Ω
- Check which Angular Velocity would break this relations –> Constraint on angular velocity
- MHD spinning NS, HMNS, differentially rotating NS $- >$ Same relations?

THE END, THANK YOU FOR ATTENTION!